APPLICATION OF QUEUEING THEORY TO A STREAM OF VEHICULAR MOVEMENT

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ABSTRACT

Traffic congestion is a growing problem in many areas in Nigeria as it increases travel time, air pollution, carbon dioxide (CO_2) emissions and fuel use because cars cannot run efficiently. This work contributes to the modeling road congestion in a sub-urban area of Ekiti State, Nigeria by way of queueing theory using stochastic process and initial value problem frame work. The approach is used to describe performance measure parameters allowing the prediction of the level of queue built up at a signalized intersection as an insight into road vehicular movement there and how such congestion occurrence can be efficiently managed.

Keywords: Queueing theory, traffic, congestion, stochastic process

1 INTRODUCTION

The role of transportation in human life cannot be overemphasized. According to Intikhab et al. (2008), efficient and effective transportation systems play vital role in taking care of the day-to-day activities in the lives of the citizens. Wane (2001) noted that transportation is an important vector for urban insertion since it gives access to economic activity, facilitates family life and helps in spinning social networks. It links the different spaces of the city on which an individual or a family has to implement his or its tri-dimensional strategy of life (i.e. family, work, residence). So, urban transportation is one of the crucial problems faced by any citizens of most urban places. Consequently, most cities around the world have witnessed increased vehicular movement during the recent century, especially since 1988 global car population exceeded 400 million (Walsh, 1990). It was noted that the reason for this phenomenon, according to Dimitriou (1990) is that in both the developed and third world countries, few activities are more poorly managed than urban transport. As such, the inadequacy of public transport to take care of the needs of commuters has intensified the demand for private cars.

2. Literature Review

Adedayo et al. (2006) stressed that many situations in life requires one to line up or queue before being attended to. Lines formed by a group of customers requiring service in front of servers are referred to as waiting lines or queues. When there is a shortfall in the service capacity compared to the demand for such service, queues are formed. Applications of queueing theory is not only limited to telephone exchange, banks and production but incudes other sectors such as, health care (Ogunlade and Okoro (2015), Olorunsola et al., (2014)). A number of studies have been conducted in Nigeria and elsewhere concerning traffic congestion and its causes. Ogunbodede (2003), in a study where GIS approach was used to investigate traffic congestion in Akure Nigeria, argued that traffic congestion is as a result of the increasing growth in motor vehicles without a corresponding improvement in transport facilities such as road network, traffic management techniques. Illegal and arbitrary roadside parking of vehicles, lack of geospatial information necessary to tackle the spatial problem, etc, were identified in the study as other causes of traffic congestion. The study further suggested the use of a dynamic Traffic Information System (TIS) structure to monitor congestions in Akure city. However, with the level of technology available today, the use of Variable Messaging Signs (VMS) located at strategic points on the road may provide a suitable alternative to the Traffic Information System. Also, the problems of intra-urban traffic in Lagos Nigeria have been studied by Bashiru and Waziri (2008). The study found that majority of commuters spend between 30 to 60 minutes on the road due to traffic congestion. They also found that the worst traffic congestion occurred on Mondays. This agrees with similar findings by Agbonika (2011) for Abuja City. Bashiru and Waziri (2008) listed the causes of traffic congestion in Lagos to include the following: Presence of pot holes/bad road, trading activities, on-street parking, loading and discharging of passengers, illegal bus stops, flooding/poor drainage, vehicle breakdown, narrow road sections, religious activities, high volume of traffic, lack of parking space and lack of traffic light at some road intersections.

Similarly, Aworemi et al (2009) studied traffic congestion in Lagos Metropolis. In agreement with Bashiru and Waziri (2008), the study noted the following major causes of traffic congestion. These include: poor road condition, inadequate road infrastructure, accident, inadequate traffic planning, drivers' behaviour and lack of integrated transport system.

Aderamo and Atomode (2011) studied the challenges faced by vehicles at road intersections in Ilorin, Nigeria. In urban areas, there are many road intersections and they are prone to traffic congestion. It was also found out in that study that traffic wardens and parking problems are part of the greatest causes of traffic delays at road intersections in Ilorin

In a study by Agbonika (2011) in Abuja Nigeria, it was found that only a few of the sampled commuting population lived within the city centre, but due to the location major government offices, with respect to the spread of residential areas, which was not properly considered in town planning caused serious delay problem. They showed that it was because of the mass movement within the same period as in the case of the government workers moving to and from work around the same period of time, that is in the morning (8.00am) and evening (6.00pm).

Road traffic management (RTM) in Ado-Ekiti, like most developing countries, has been very challenging and most attempts to address the problem have yielded little results. This includes arterial routes expansion to accommodate increased vehicular traffic volume, mass transport facility and road junction control measures among others. Given the factors fuelling vehicular population in the capital, Ado-Ekiti roads appear woefully inadequate to match present vehicular population on our roads. This disparity is causing road traffic congestion, which is crippling mobility in the capital and gradually grinding economic activities in the city to a halt. Road traffic signal used as a road junction control measure to minimize congestion currently is inadequate. This is evident from the observation that although the majority of the signal lights have been revamped, they are being augmented currently by human traffic wardens (TW) to manage the situation. It has been observed on several occasions that the activities of these wardens on traffic flow rates at signalized junctions during peak hours and their interventions amazingly make positive impact on the congestion problem. Clearly automatic signal control approach to traffic management is not working as expected and delays due to road congestion and its accompanying cost is weakening the economic impact of urban road transport. Given that roads are constantly used by vehicles at all times, human intervention for optimal road vehicular traffic control by Traffic Wardens cannot be avoided, especially for developing economies like Nigeria. Also given that the disparity in road expansion and road vehicular population increases, the positive impact of Traffic Wardens on road traffic congestions is likely to be inefficient if traffic flow and evolution, especially for the unexpected congestions is not accurately predicted.

However relatively little attention, by way of research has been given to this congestion problem in the urban area of Ado-Ekiti. A scientific approach investigation of the problem could encourage the improvement of transport policies and strategies in place to mitigate this economic debilitating spate of congestion on our roads. For instance as a first step towards prescribing a solution to the problem, the ability to forecast traffic volumes for any time period especially those critical periods cannot be over emphasized. This forecast approach also has the capacity to directly support proactive traffic control including educated deployment of Traffic Warden to critical areas as well as accurate travel time estimations.

3. THE M/M/1 QUEUEING MODEL

Suppose that customers arrive at a single server service station in accordance with a Poisson process having rate λ. That is the time between successive arrivals are independent exponential random variables having mean $1/\lambda$. Each customer, upon arrival goes directly into service if the server is free and if not, the customer joins the queue when the server finishes serving a customer, the customer leaves the system and the next customer in line (if any) enter service. The successive service times are assumed to be independent exponential random variables having mean $1/\mu$. Let P_n be the long run probability that the system contains exactly n customers then (Bunday;1986),

$$P_n = \rho^n(1-\rho), \qquad n \geq 1 \qquad (1)$$

$$\rho = \frac{\lambda}{\mu} < 1 \textit{ is known as traffic intensity}$$

The average number of customers in the system clearly is given by

$$E(N) = \sum_{n=0}^{\infty} nP_n = \frac{\lambda}{\mu - \lambda} \qquad (2)$$

From Little's formular $E(N) = \lambda E(T)$, then the average waiting time is

$$E(T) = \frac{E(N)}{\lambda} = \frac{1}{\mu - \lambda} \qquad (3)$$

Let T_Q denote the amount of time that a customer spends waiting in queue and S the amount of time a customer spends in service, then average waiting time in the queue is

$$E(T_Q) = E(T) - E(S) = \frac{\lambda}{\mu(\mu - \lambda)} \quad (4)$$

And let N_Q be number of customers waiting in the queue then, the average number of customers waiting in the queue is

$$E(N_Q) = \lambda E(T_Q) = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad (5)$$

4. COMPUTATION OF QUEUEING PARAMETER

We consider the traffic stream of vehicular movement and in this case the road vehicles constitute our arrivals process. It is assumed that the time interval between successive arrivals and serving time is independent and identical distributed. In any sufficient interval of time at most only one arrival can occur. The system is also assumed to reach a steady state, a condition that the rate of arrival and service are constant. The queueing discipline observed was first-come-first-served (FCFS). Observation at this intersection was made for thirty days(30) days during which the arrivals were noted at morning rush hours every five (5) minutes from 7:30 hours to 8:30 hours. The analysis of data collected would be carried out using the single server system formula.

Performance Measure Of The System For Vehicles Negotiating The Ring

Arrival Rate $\lambda = 17 cars$ per minute, Service Rate $\mu = 19$ cars per minute, hence traffic Intensity $\rho = \frac{\lambda}{\mu} = 0.8947$.

$$\text{Average number of vehicles in the system } E(N) = \frac{\lambda}{\mu - \lambda} = \frac{17}{19 - 17} = 9 \; cars$$

$$\text{Average time cars spent in the system } E(T) = \frac{1}{\mu - \lambda} = \frac{1}{19 - 17} 0.5 \; minute$$

Average time waiting in the queue $E(T_Q) = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{17}{19(19-17)} = 0.4474\ minute$

Average number of cars waiting in the queue $E(N_Q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(17)^2}{19(19-17)} \approx 8\ cars$

Performance Measure Of The System For Vehicles Negotiating The Bend

$Arrival\ Rate\ \ \lambda = 18\ cars\ per\ minute$

$Service\ Rate\ \mu = 21\ Veh.per\ minute$

$Traffic\ Intensity\ \ \rho = 0.8571$

$Average\ number\ of\ vehicles\ in\ the\ system\ E(N) = \frac{18}{21-18} = 6\ cars$

Average time a vehicle spent in the system $E(T) = \frac{1}{21-18} = 0.3333\ min$

Average time waiting in the queue $E(T_Q) = \frac{18}{21(21-18)} = 0.2857\ min$

Average number of customers waiting in the queue $E(N_Q) = \frac{(18)^2}{21(21-18)} = 5.1429\ cars$

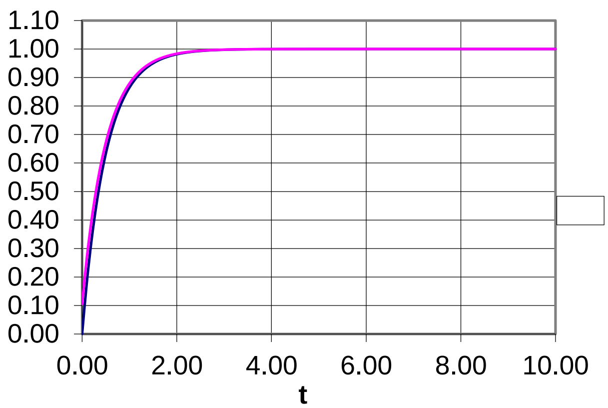

Fig.1. Waiting time distribution for vehicles negotiating the ring

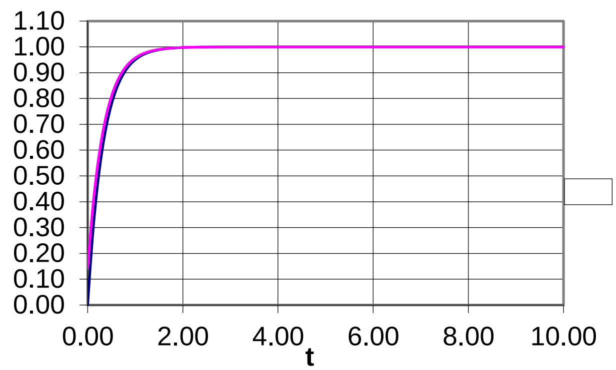

Fig 2. Waiting time distribution for vehicles negotiating the bend

The stream of traffic selected for this study used the single server model M/M/1. But for efficiency purposes, it is worth to assume a multi-server model being used by the same store so as to compare the efficiency levels. To compare, the study has proposed an M/M/2 model. The efficiency parameters for the two models are shown in Table 1. The traffic intensity has changed significantly from 89.47% to 44.47%. This shows that as more lanes are introduced, then there will be less congestion.

Table 1: Efficiency parameters under two different queueing models

Parameters	M/M/1	M/M/2
λ	17	17
μ	19	19
ρ	0.8947	0.4447
$E(N)$	9	1.11
$\mathrm{E(T)}$	0.5	0.0658
$\mathrm{E(T_Q)}$	0.4474	0.013169
$\mathrm{E(N_Q)}$	8	0.022

5. CONCLUSION

This work measured the traffic flow at a junction in a sub-urban road during the morning rush hours and have demonstrated features of the queue built up at the signalised intersection with data and modeled the traffic flow there as a M/M/1. We have shown that the current queue system will continue to develop heavy traffic, evident by the growing queue length and waiting time, during the peak hours. This obviously has quality of service implications for the system at the moment evident from the traffic intensity estimates from the data collected. This work therefore gives insight into possible undesirable levels of vehicular traffic congestion and the obvious question is how to minimise or at least contain these undesirable levels in order to optimise waiting time at the intersection. Possible line of attack is an increase in the road infrastructure.

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